SRIC FOUNDATION
SWISS RESEARCH INSTITUTE ON COMMODITIES

1-6-2016
WORKING PAPER SERIES

---- PUBLICATION RESEARCH PAPER ----

The Valuation of Hydro Virtual Pump Storage Contracts

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1 INTRODUCTION

1.1 Context

The liberalization of power markets has led producers and wholesalers to be faced with a new class of optimization problems due to the arising of spot markets where electricity is traded on a daily basis. New types of power contracts have been developed by energy companies in order to tackle such problems and enable them to hedge the new risks they face.

Tolling agreements enable power marketers to own an asset and thus reduce trading risk by allowing asset-backed trading. Besides, energy marketers are interested in these contracts as they provide them with flexibility and because of the important upside profit. If the agreements are based on a physical asset, they are also attractive to the power plant owners mainly because they free themselves from price risk and enable them to focus on maintaining the physical hardware of their plants and leave the marketing of the power to other players.

These agreements can be financial or virtual. In such case there are no real assets backing the contracts and these are called Virtual Power Plants (VPPs). VPPs give the marketers the option to optimize a fictitious power plant without the technical risks and operational matters involved in running a real-life plant. VPPs are also attractive to plant owners in order to balance out the fuel exposure of their assets and thus in order to diversify their asset portfolio.

The aim of this thesis is to introduce tolling agreements and the theoretical framework of the different pricing techniques used to value them. On the practical side, the main objective of is to build an Hourly Price Forward Curve to be used in the contract evaluation and to evaluate the intrinsic value of the contract.

1.2 Tolling agreements

In the electric sector, a tolling agreement is an agreement concluded between a buyer (a power marketer) and a power producer where the power marketer pays a fee or toll to the producer to convert natural gas (or another fuel) into electricity. Depending on the agreement, the buyer will have the right to either operate the power plant or simply take the output electricity during pre-specified time periods subject to certain constraints.

In addition to inherent operational constraints of the power plant, usually other contractual limitations are included in the contract on how to operate the plant or how much electricity can be demanded.

VPPs are purely financial contracts and they are based on the operation of a fictitious power plant. Most VPPs include constraints very similar to those of real power plants like for example a limit on the number of start-ups.

‘A Virtual Pump Storage (VPS) attempts to replicate financial flexibility of a pump-hydro storage plant, giving the owner the possibility of calling power previously stored for sale in the spot market or of putting power into storage for future delivery’ [Fanone].

These contracts are also called Hydro tolling agreements.

1.3 Valuation of Tolling agreements

In order to evaluate the economics of the agreement, not only the positive and negative revenue potential of the agreement must be understood, but also the opportunities of hedging risk and locking in value.

The terms and conditions that are specified in the contract enable the marketer to evaluate from the economic perspective the opportunity. Some of the value of a tolling agreement is known for sure and is equal to its *intrinsic value*. The intrinsic value is the value of the agreement if all the operational decisions are taken immediately and profits are locked in. This can be achieved by buying and selling the electricity forward. The intrinsic value is the lower bound of the contract’s value.

However, the contract has more value than its intrinsic value and this component is called the *extrinsic value*. The extrinsic value is the value given to the flexibility that the marketer receives from the optionality of the contract and the flexibility of being able to exercise the contract only in hours which will provide him with a positive margin.

The total value of a contract is therefore equal to the sum of its intrinsic value and its extrinsic value.

TOTAL VALUE = INTRINSIC VALUE + EXTRINSIC VALUE

From a financial point of view, virtual assets can be assimilated to an American option portfolio. The valuation of a hydro tolling contract is similar to the evaluation of a series of call and put options. The use of real option theory is thus a way of determining the extrinsic value of the contracts.

The four most used valuation methodologies are: (1) rolling intrinsic approach, where given the current forward prices, the intrinsic value of the contract is locked in, (2) stochastic optimization through the use of a binomial tree, (3) stochastic optimization using Monte Carlo simulations and (4) application of real option theory and solving a partial differential equation.

2 PROBLEM SET UP: TERM SHEET OF A VIRTUAL PUMP STORAGE

The following Term Sheet of a VPS contract is analysed. The VPS is offered for the year 2015, with the Italian market as a reference. The main parameters of the contract are summarised in the following table.

Term / Definition	**Explanation**
Description of the contract	The buyer buys from the seller a Virtual Pump Storage Contract (VPS) with physical delivery in Italy in return for monthly capacity payments. The VPS contract gives the buyer the right to deliver hourly profiles to the seller (Pump Energy) and in exchange receive hourly profiles from the seller (Turbine Energy).
Start Date	01.01.2015 00:00
End Date	31.12.2015 24:00
Underlying Commodity	Italian Power settled at Prezzo Unico Nazionale (PUN)
Nominal Capacity	Turbine: 50 MW; Pump: 50 MW
Pump Efficiency (E)	0.70
Capacity Schedule	The capacity nominated for Pump and Turbine Nomination for each hour must be a whole number of MW between 0 MW and 50 MW.
Reservoir Level (RL)	If the Reservoir Level at hour t exceeds the Reservoir Level constraints ($0 \leq RL_t \leq Max$), then the Reservoir Level at hour t is assumed to be equal to the constraint. Nomination volumes outside of these constraints will not affect the Reservoir Level and are subject to the process detailed in Infringement of Constraints.
Max. Reservoir Level	3,000 MWh
Reservoir Level at Start	0 MWh
Reservoir Level at End	0 MWh
Infringement of Constraints	If the buyer nominates a schedule that causes the Reservoir Level to fall below 0 MWh or go above 3,000 MWh, the seller will charge the buyer the PUN price of the hour with a premium of €200 per MWh for each MWh outside of the Reservoir Level constraints.

3 HOURLY PRICE FORWARD CURVE

The first step in order to evaluate the VPS contract is to set up the underlying model and simulate the hourly prices during the span of the contract. These prices are obtained through an Hourly Price Forward Curve (HPFC).

The valuation of the contract against the HPFC serves as the deterministic evaluation of the contract. This is done by calculating an optimal dispatch of the contract using the HPFC as price signal.

It is important to note that the deterministic evaluation is only the lower bound of the valuation (the intrinsic value) as it does not consider the optionality of the contract.

3.1 Theoretical background

3.1.1 Introduction to Hourly Price Forward Curves

'The purpose of the Hourly Price Forward Curve (HPFC) is to calculate an arbitrage-free average price profile at an hourly resolution. The hourly basis is the standard resolution of most electricity markets for the day-ahead spot price in contrast with the load frequency, which is on 15 minute basis. The idea of the HPFC is an hourly prediction of the average price based on normal weather and load information. In contrast to a spot forecast, the HPFC does not usually consider extreme weather events or spikes but focuses on normal weather and structural market conditions' [Hildmann]. In the market, the most widely used method by all market participants for arbitrage free hourly pricing of electricity contracts is the HPFC.

Electric prices display seasonality at the yearly, monthly, weekly and intra-day level, following a relatively predictable pattern. This predictability of patterns is the corner stone of a forward curve builder. The HPFC shapes the forward curve based on historical data and price differences (shapes) between the different hours of the day, the different days of the week and the different months of a year.

3.1.2 Construction Method

The HPFC is an hourly price profile which is artificially constructed as an arbitrage free equivalent to the liquid exchange-traded futures products. One of the key assumptions that must hold in order to guarantee the arbitrage free condition is that the average of all the hourly prices of an HPFC in the period of a futures product must be equal to the futures product price:

$$\frac{1}{T}\sum_{t=1}^{T} HPFC_t = F$$

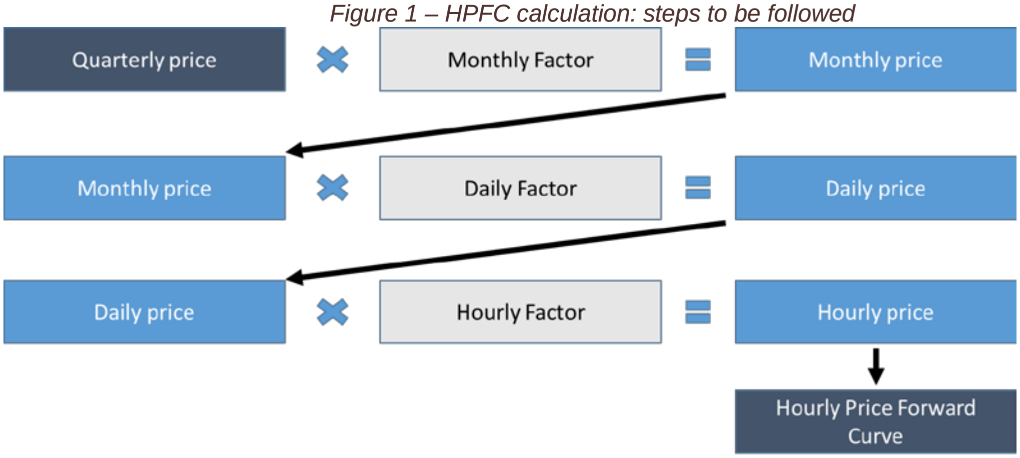
Where t is the time index, T is the maturity, $HPFC_t$ is the hourly price at time t and F is the future product.

The procedure used to build an HPFC usually consists of 2 steps:

1. Estimation of an hourly profile through the determination of monthly, weekly and daily factors
2. Input of the futures product prices in to the hourly profile

Every hour in the HPFC is given a weight obtained from the estimated hourly profiles. The profiles only contain the weight information; the price level is given by the futures product price inputted in the HPFC. The general steps for calculating the HPFC are summarised in the following figure where it can be seen that the input of the model is the quarterly futures product price and the output is the HPFC composed of hourly prices.

Figure 1 – HPFC calculation: steps to be followed



The hourly profiles are obtained by combining different shape factors, namely monthly, daily and hourly factors.

'There are generally three ways to estimate the shape of the hourly profile:

1) statistical model: calculation of the profile based on average calculations of historical spot price time series.
2) fundamental model: calculation of the profile based on supply and demand curve (i.e merit order curve & load).
3) combined model: combination of the two models to calculate the hourly profile. Also known as hybrid model.

While the combined model and the fundamental model use the underlying physical relationships for the estimation, the necessary information is considerable and also not always available. Statistical models are less data dependent but react more slowly to fundamental market changes' [Hildmann].

3.2 Construction

3.2.1 Applied methodology

In order to analyse the contract described in the previous section, an HPFC was built using the statistical methodology approach. The span of the HPFC is one year, covering all the hours of 2015.

The database used is the hourly Italian Prezzo Unico Nazionale (PUN) prices from January 2011 to June 2014 (equivalent of 3 years and a half of historic data). The futures products used as inputs of the model are the forward base and peak quarter contracts.

The hourly profiles are derived through the calculation of monthly factors, daily factors and hourly factors. Two parallel processes are run, one for base prices and one for peak prices. Thus, a monthly base factor is determined as well as a monthly peak factor. The same applies to daily factors, both a base and peak daily factors are determined.

Regarding the daily factors, 7 day types (Monday to Sunday) plus holidays are defined.

A set of weighing factors, w_{year}, are applied to each year of data in the database in order to give more weight to recent years. These weighing factors have been introduced to take into account that in certain electric markets the shape of the hourly price curve has changed significantly in recent years due to the introduction of renewables in the stack. Such is the case of the Italian market which has experienced a high decline of peak hour prices due to the high penetration of solar energy leading to a much flatter daily shape of the curve.

The methodology used to construct the HPFC is summarised in the following table:

Table 1 - HPFC methodology

Price basis	3 prices are considered: Peak, Off peak, Base weekend
Weighing of years	Different weights are applied to the years in the dataset (more weight given to recent years) in order to incorporate the effect of renewable penetration and other determining factors that have shaped the current stack.
Database	3 and a half years of PUN spot prices are considered
Data history	01.01.2011 – 30.06.2014
Day types	7+1 day types are defined within the typical week: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday and Holidays
Holidays	The selected Italian holidays are: New Year's Day, Epiphany, Easter Monday, Labour Day, Republic Day, Assumption Day, All Saints Day, Immaculate Conception Day, Christmas Day, St Stephen's Day
Outliers/Data exclusion	The factors are calculated for each year in the dataset and then are averaged out. No factors have been taken out of the sample.

Normalisation of the factors	The factors are normalised to sum to 1 to make them arbitrage free. The adjustments of the factors is of less than 2%.

3.2.2 Monthly factors

The inputs of the model are the forward quarter prices. The first step is to transform these prices into monthly prices through the application of a monthly shape factor.

The monthly shape factor is calculated as follows:

$$M_{y,m} = \frac{\sum_d \sum_h P_{y,m,d,h}}{d_m * 24}$$

Where $P_{y,m,d,h}$ is the price in hour h, day d, month m and year y and d_m is the numbers of day in month m.

Secondly a weighted average of the monthly factor is then calculated using the weights applied to each year. The formula used is:

$$M_m = \frac{\sum_y (M_{y,m} * w_y)}{\sum_y w_y}$$

Four different factors $M_{y,1}$ are obtained; each for the 4 years of historic data used in the simulation: $M_{2011,1}$, $M_{2012,1}$, $M_{2013,1}$ and $M_{2014,1}$.

Then the weights are applied to each year:

$$M_1 = \frac{\sum_y (M_{y,1} * w_y)}{\sum_y w_y} = \frac{M_{2011,1} * w_{2011} + M_{2012,1} * w_{2012} + M_{2013,1} * w_{2013} + M_{2014,1} * w_{2014}}{w_{2011} + w_{2012} + w_{2013} + w_{2014}}$$

3.2.3 Daily factors

Once the shape of each month with respect to the quarter is determined, the following step is to determine the shape of each day of the week. A different week shape is calculated for each month. Holidays are excluded in the calculations of daily shape factors as they are treated separately due to their particularities.

For every given day of a week a shape factor is calculated. First a daily factor is estimated for every single day in the historic data set:

$$D_{y,m,d} = \frac{\sum_h P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}}$$

Secondly a week shape factor is derived from the above daily factors as follows:

$$W_{y,m,d} = \frac{\sum_d D_{y,m,d}}{d_m^d}$$

Where d_m^d is the number of days d (ie. Mondays, Tuesdays etc.) in month m excluding holidays.

Finally a weighted average of the weekly factor is then calculated using the weights applied to each year. The formula used is:

$$W_{m,d} = \frac{\sum_y \left(W_{y,m,d} * w_y\right)}{\sum_y w_y}$$

A similar process is followed to estimate peak weekly factors, the only difference being the estimation is done based on the peak hours. Thus the day peak factor is equal to:

$$D_{y,m,d}^{Peak} = \frac{\sum_{h=9}^{20} P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}}$$

The rest of the formulas are identical.

3.2.4 Hourly factors

Once the shape of each day in a week is determined, the final step is to estimate the hourly shape of every day. Peak hours and off-peak hours are determined separately using the respective peak or base factors, and the hourly shape of holidays is determined separately using the specific daily shape of holidays.

First an hourly factor is estimated. Depending on whether the hour considered is a peak or an off peak hour, the formulas applied are different. For off-peak hours the factor is given by:

$$H_{y,m,d,h} = \frac{\sum_d \frac{P_{y,m,d,h}}{D_{y,m,d}^{Off-peak}}}{d_m^d}$$

And for peak hours the factor is given by:

$$H_{y,m,d,h} = \frac{\sum_d \frac{P_{y,m,d,h}}{D_{y,m,d}^{Peak}}}{d_m^d}$$

Saturdays and Sundays are treated separately and the hourly factors are computed using the base price:

$$H_{y,m,d,h} = \frac{\sum_d \frac{P_{y,m,d,h}}{D_{y,m,d}^{Base}}}{d_m^d}$$

Where:

$D_{y,m,d}^{Base} = \frac{\sum_{h=1}^{24} P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}}$, $D_{y,m,d}^{Peak} = \frac{\sum_{h=9}^{20} P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}}$ (as previously defined), $D_{y,m,d}^{Off-peak} = \frac{\sum_{h=1}^{8} P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}} + \frac{\sum_{h=21}^{24} P_{y,m,d,h}}{\sum_d \sum_h P_{y,m,d,h}}$

And where, d_m^d is the number of days d (ie. Mondays, Tuesdays etc.) in month m excluding holidays.

Finally a weighted average of the hourly factor is then calculated using the weights applied to each year. The formula used is:

$$H_{m,d,h} = \frac{\sum_y \left(H_{y,m,d,h} * w_y\right)}{\sum_y w_y}$$

3.2.5 Profile construction

Finally once all the shape factors are obtained, the HPFC is constructed by applying the factors to quarterly futures product prices inputted in the model.

First the base, peak and off peak prices of each day are calculated as follows. The Base price is equal to:

$$B = Q_q^{Base} * M_m^{Base} * W_{m,d}^{Base}$$

The peak price is calculated for weekdays only using the following formula:

$$Pk = Q_q^{Peak} * M_m^{Peak} * W_{m,d}^{Peak}$$

In case a holiday day falls during a weekday, the peak of the holiday is determined as a function of the base price and a factor which determines the average peak to base ratio of holidays:

$$Pk = B * f^{peak}$$

Where f^{peak} is the average peak to base ratio of all holiday days included in the historic dataset.

Finally the off peak price is derived from the base and peak prices as follows:

$$OP = 2 * Pk - B$$

Once the base, peak and off-peak prices of each day are known, the hourly prices are calculated using the hourly shape factors.

Peak hours of weekdays: $H = Pk * H_{m,d,h}$

Off-peak hours of weekdays: $H = OP * H_{m,d,h}$

Saturdays and Sundays: $H = B * H_{m,d,h}$

3.3 Implementation

3.3.1 Structure

The HPFC has been built in Excel. This choice is based on the fact that Excel is a program know by most people in the industry and very accessible making it an obvious choice for the development of the HPFC.

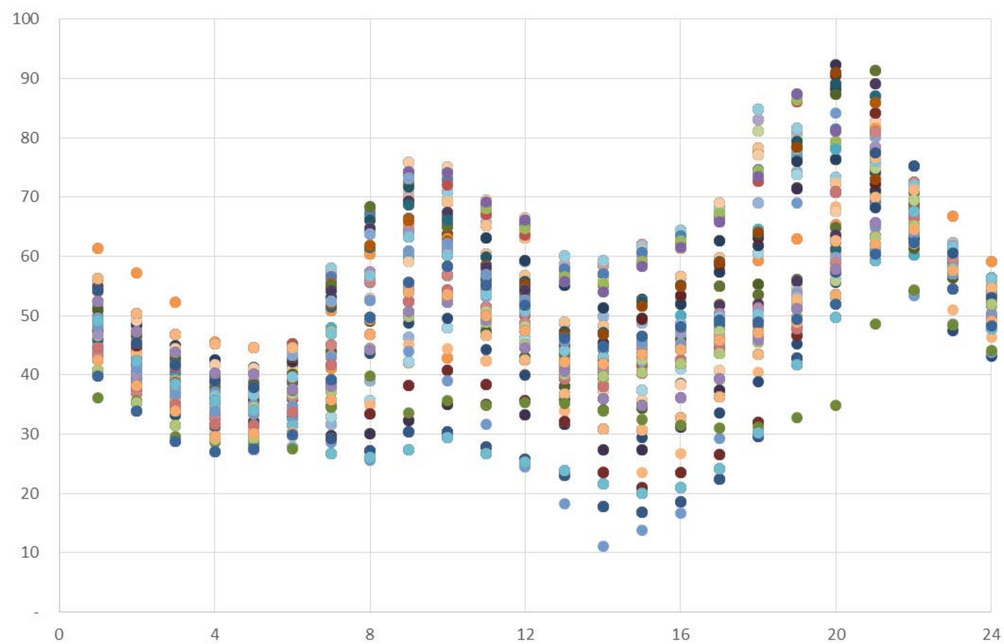
The HPFC was developed in the most generic and flexible manner. The user of the HPFC can adapt it to fit its particular problem; indeed the HPFC can be easily applied to different markets, the data set and the data history can be changed, the time length of the HPFC can be reduced and the list of holidays can easily be amended. This makes the HPFC are very flexible tool that can be used in the valuation process of any contract and that needs only small changes which are simple and quick to implement.

3.3.2 Numerical results

The quarterly futures product prices traded in the market are used as inputs of the HPFC to determine the price level. This approach is a deterministic approach where one price scenario is determined and later optimised. The advantage of this approach is its simplicity and the fact that by using current forward prices, the contract can be hedged and thus traders can lock in the costs/profits of the tolling agreement.

The Term Sheet being studied in this thesis was analysed on the first of July 2014 and an HPFC for 2015 was obtained. The following figure shows the obtained hourly price distribution:

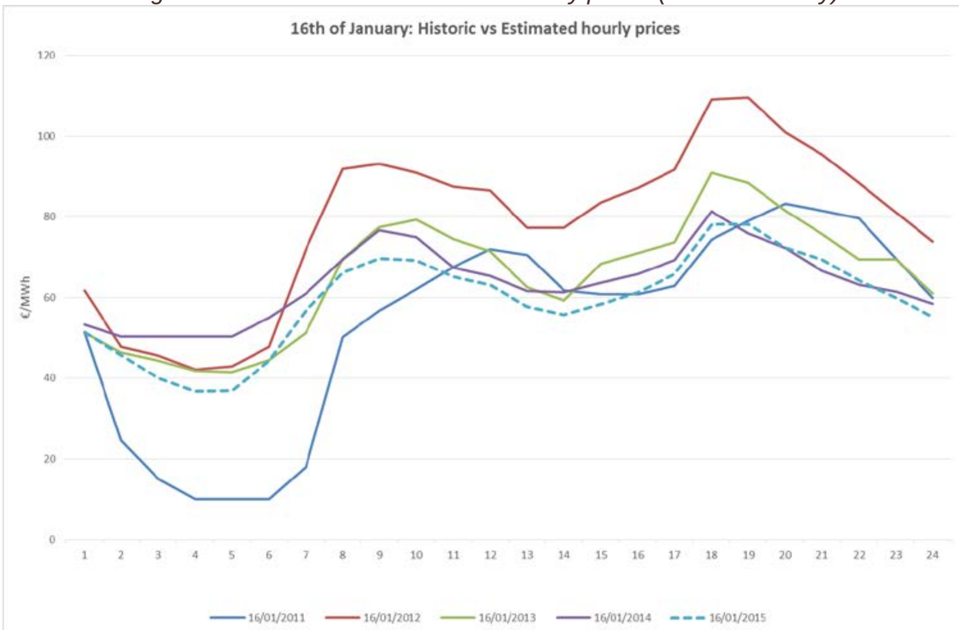
Figure 2 - Hourly Price distribution



The first and last hours of the day, hours 1 to 7 and 22 to 24, have a smaller price distribution whilst peak hours experience a broader price distribution. Most of the outliers that can be observed in the price distribution correspond to holidays.

Finally, analysing a particular day of the year, for example the 16th of January 2015 (a Friday), the following hourly price distribution is obtained:

Figure 3 - Historic versus estimated hourly prices (16th of January)



As it can be observed the shape of the forecasted prices is very much aligned with the historic shapes. The only year which experiences much lower off peak prices is the 16th of January 2011 as it is a Sunday.

3.3.3 HPFC back testing

The results obtained from the HPFC were back tested using historical data and comparing results with actual spot prices. In particular, the back test was carried out by comparing the shape of the hourly prices obtained from the HPFC model with the actual shape of spot prices. Once all the inputs were entered into the HPFC model, the obtained hourly prices for the first two quarters of 2014 were compared to the actual spot prices of 2014 and an error matrix was built. In the following table are summarised the average errors obtained by quarter and by month, the values highlighted in blue are error values superior to 10%:

Table 2 – Back testing error matrix [€/MWh]

Period	Base	Peak	Peak-WD	Off-Peak	Pk/Base
Q1	-0.21	-0.21	-0.17	-0.20	-0.01
Q2	0.01	-0.06	-0.05	0.08	-0.06
Jan	-0.09	-0.07	-0.06	-0.11	0.02
Feb	-0.25	-0.26	-0.22	-0.24	-0.02
Mar	-0.28	-0.31	-0.24	-0.25	-0.05
Apr	-0.01	-0.05	-0.03	0.04	-0.05
May	0.02	-0.08	-0.07	0.14	-0.11
Jun	0.01	-0.03	-0.03	0.06	-0.04

The first noticeable result is the difference in results between the first and second quarter of 2014. Indeed whilst the average errors observed in the second quarter are quite low, in the first quarter of the year there are differences of 20% between the prices obtained from the HPFC model and the actual spots prices. In particular, the months which show the highest error deviation are the months of February and of March.

In the second quarter although all the monthly errors are small, it is of particular interest to analyse the error seen in the month of May in the Peak to base ratio. This ratio is very important in VPS contracts and getting the number correctly is of key importance thus an error of 11% would not be acceptable.

In order to understand the reason behind these errors one must study the differences between the forward prices and the actual delivery prices (or spot prices). By comparing the two it is possible to discern whether the differences seen between the spot prices obtained from HPFC model and the actual spot prices are due to an error in the HPFC model or whether the differences are in part due to the fact that the forward contracts were not well priced and the delivery levels were actually very different to the forward contract trading levels.

The following tables show, for both for the base and the peak contracts, the error between delivery prices and forward prices. The forward prices used are the last traded prices for a given contract. In order to make the below comparison meaningful, the futures products used as inputs of the HPFC in the back testing have the same time-to-maturity intervals.

Table 3 - Error between base historical forward prices and actual delivery prices

Delivery versus Forward

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2010	-0.02	0.00	0.05	0.03	-0.04	-0.06	0.03	0.05	-0.07	0.00	-0.07	0.01
2011	-0.04	0.02	0.00	-0.03	0.06	-0.04	-0.08	0.05	0.08	0.01	-0.02	-0.02
2012	0.01	0.11	-0.04	-0.03	-0.08	0.05	0.01	0.10	-0.06	-0.10	-0.08	-0.02
2013	-0.08	-0.01	0.06	-0.09	-0.08	-0.05	0.02	-0.01	0.01	0.02	-0.01	0.06
2014	-0.11	-0.15	-0.06	0.01	0.05	-0.02						

Table 4 - Error between peak historical forward prices and actual delivery prices

Delivery versus Forward

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2010	-0.05	-0.04	0.03	-0.04	0.01	-0.06	0.05	0.03	-0.11	0.03	-0.05	0.00
2011	-0.07	0.05	0.01	-0.03	0.05	-0.05	-0.09	0.01	0.13	0.04	0.03	0.02
2012	0.03	0.20	0.00	-0.06	-0.07	0.09	0.02	0.05	-0.02	-0.06	-0.02	0.01
2013	-0.07	-0.04	0.06	-0.07	-0.01	-0.06	-0.03	-0.04	-0.02	0.02	-0.01	0.04
2014	-0.10	-0.10	-0.06	-0.04	-0.25	0.03						

The above analysis shows that during the first quarter of 2014 the actual spot prices diverged by over 10% from the forward value on average. Indeed the first quarter of 2014 was a very particular period in the Italian market as prices fell significantly and in particular very large differences were observed between the levels at which the futures products were trading at and the actual level at which this contracts delivered. This particularity of the Italian market in the first quarter of 2014 largely explains the errors observed in the back testing error matrix.

During the second quarter of 2014, the differences between the forward contracts and the actual delivery prices returned to normal levels as per the historical data. The only noticeable number is the peak contract of May. The peak spot prices of May were 25% lower that the trading level of the May 2014 Peak contract. This large mispricing of the forward contract explains the error of 11% seen in the peak to base ratio of May obtained from the HPFC model.

Considering the above observations which largely explain some of the large errors seen in the back testing error matrix, one can conclude that the results obtained from the HPFC model are very good and that the observed error levels are very much in line with those observed in the market between forward prices and actual spot prices. In particular the HPFC model seems to perform quite well with getting a correct peak to base ratio. This is a very good result as getting an accurate peak to base ratio is of key importance when assessing VPS contracts.

4 INTRINSIC VALUE: OPTIMIZATION MODEL

The Optimization model developed and implemented in this section is based on a particular type of tolling agreement which is the financial agreement. These are less complex than physical contracts and the operational risks are borne by the plant owner. This is why they are the ones trading firms typically enter into.

4.1 Model formulation

The VPS contract presented in the Term Sheet of the previous section can be modelled through the following model:

Objective function:

$$\max \sum_{i=1}^{T} (Qt_i - Qp_i) * p_i - L_i * (R_i - R_{Max}) * (p_i + p_{pen})$$

Subject to:

$$R_i = R_o + \sum_{j=1}^{i} E * Qp_j - Qt_j$$

$$0 \leq Qt_i \leq Qt_{Max}$$

$$-Qp_{Max} \leq Qp_i \leq 0$$

$$\begin{cases} If\ 0 \leq R_i < R_{Max}\ then\ L_i = 0 \\ Otherwise\ L_i = 1 \end{cases}$$

Where

p_i is the price in hour i, €/MWh

Qt_i is the turbined quantity in hour i, MWh

Qt_{Max} is the maximum quantity that can be turbined in any given hour, MWh

Qp_i is the pumped quantity in hour i, MWh

Qp_{Max} is the maximum quantity that can be pumped in any given hour, MWh

R_i is the reservoir level in hour i, MWh

R_{Max} is the maximum reservoir level, MWh

R_o is the initial reservoir level in hour 0, MWh

E is the pump's efficiency

p_{pen} is the penalty price to be paid for exceeding the maximum reservoir level, €/MWh

L_i is an indicator that the reservoir level is above the maximum level in hour i

T, is the time span of the contract, hours

The restrictions defined previously are:

1. The reservoir level equation which guarantees that the level of the reservoir is coherent with the turbining and pumping nominations:

$$R_i = R_o + \sum_{1}^{i} E * Qp_i - Qt_i = 0 + \sum_{1}^{i} 0.7 * Qp_i - Qt_i$$

2. The turbining and pumping nomination constraints that guarantee the correctness of nominations:

$$0 \leq Qt_i \leq 50 \text{ and } -50 \leq Qp_i \leq 0$$

3. The maximum reservoir level breach indicator:

$$\text{If } R_i \geq R_{Max} \text{ then } L_i = 1 \text{ and if } R_i < R_{Max} \text{ then } L_i = 0$$

Other parameters defined in the Term Sheet are the following ones:

Final reservoir level, R_{8760} is: $R_{8760} = 0$

The maximal reservoir level is: $R_{Max} = 3000$

The penalty to be paid for exceeding the reservoir level is: $p_{pen} = 200$

4.2 Model implementation and results

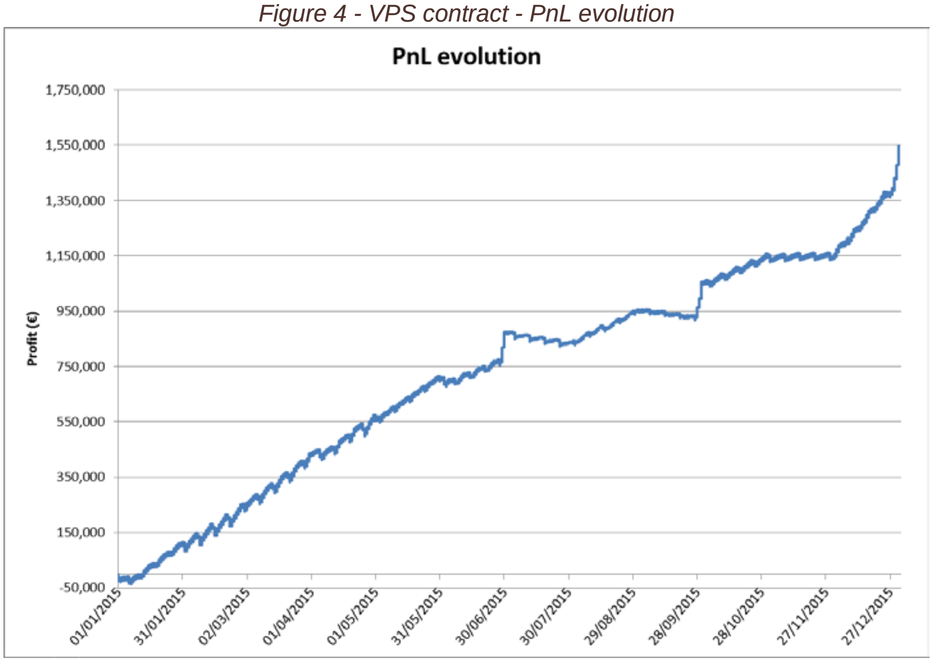
The optimisation problem was solved at hourly granularity using the OpenSolver software. OpenSolver is an Open Source optimizer for Microsoft Excel. It is 'an Excel VBA add-in that extends Excel's built-in Solver with a more powerful and faster solver allowing to solve larger models' [OpenSolver]. One of the major advantage of this program with respect to other optimisers is its ease of use and the fact that it is compatible with existing Excel Solver models as it directly uses Excel. This is an important advantage as it makes it user-friendly on a trading desk environment were all users have at least a user level knowledge of Excel. The optimisation problem has been modelled on a separate Excel file which is linked to the HPFC enabling a direct input of the hourly prices used as the deterministic scenario.

The optimal dispatch of the VPS contract results in a Profit of 1,547,421.94€ which is equal to the deterministic value of the contract, that is the lower bound of the contract's value.

The obtained value is the Model's PnL, which is of course not a guarantee of profit once the contract is actually implemented. There are some model risks to be taken into account. Probably the most important risk is the hourly prices obtained from the shaping done by the HPFC model to the curve. Indeed, depending on the hourly prices used to run the optimisation model, the obtained profit could differ significantly. This is one of the reason behind the importance of having an HPFC model that can accurately shape the curve.

The evolution of the Model's PnL is represented in the following graph:

Figure 4 - VPS contract - PnL evolution



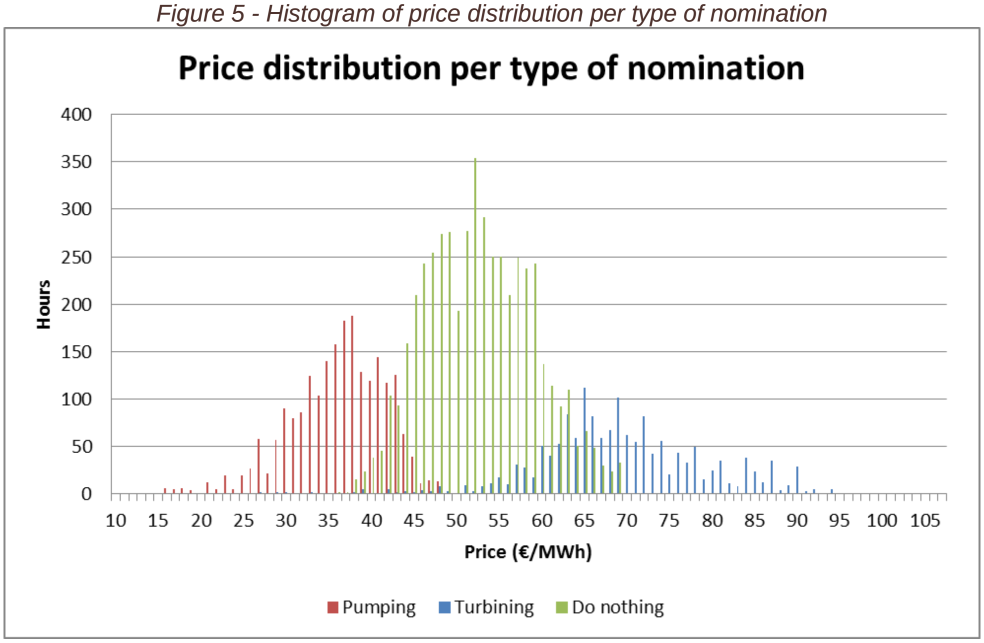
The PnL increase seems to be constant although a slow-down is noticed during the months of July, August and September. One of the reasons behind this slowdown in the third quarter is the higher solar generation being more intense the summer months (July and August) and leading to a lower peak to base ratio and thus to a lower spread between pumping hours and turbining hours. Thus, the profit that can be made is much lower and as a consequence the number of hours during which the nomination is to turbine or pump is much lower as well. The following table summarises the number of turbining, pumping and do nothing hours per quarter.

Table 5 - Quarterly nomination distribution

Period	Pump	Turbine	Do nothing	Hours
Q1-15	669	468	1,023	2,160
Q2-15	694	487	1,003	2,184
Q3-15	312	219	1,677	2,208
Q4-15	591	414	1,203	2,208

The relationship between the nomination type (turbining nomination, pumping nomination and do nothing nomination) and the price levels is also important to analyse. The following histogram reflects the price distribution per nomination type:

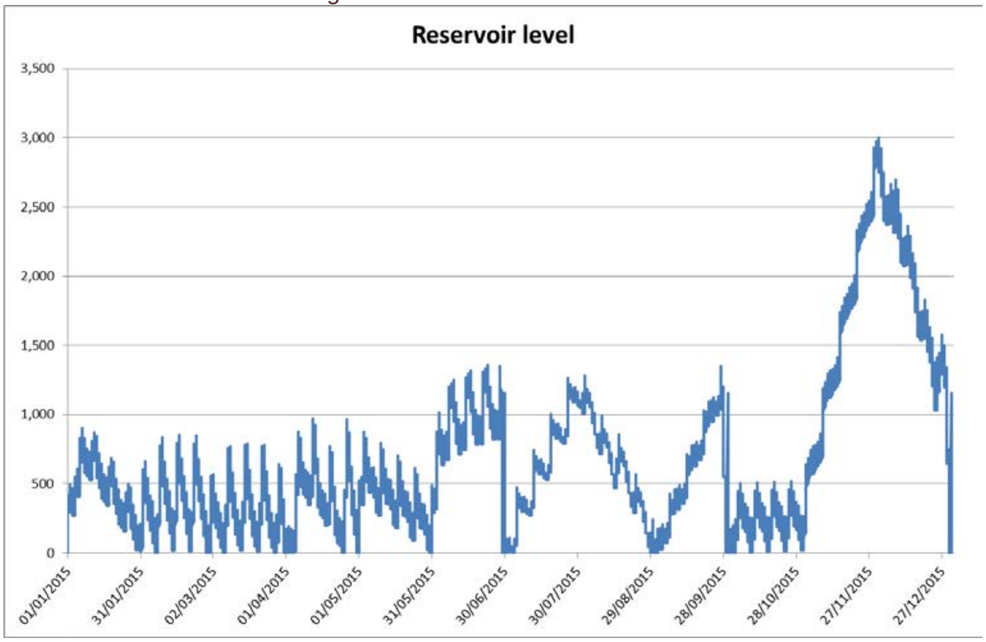
Figure 5 - Histogram of price distribution per type of nomination



Most of the pumping nominations take place at low price levels whilst the turbining nominations take place at high price levels. Do nothing nominations occur in middle range prices. This explains the jumps that can be observed in the PnL evolution graph. At weekends as prices are lower there is an accumulation of pumping hours (resulting in a PnL loss) whereas during week days, prices increase due to the increase in demand, and so turbining hours are more common resulting in an increase in PnL.

Finally the following figure illustrates the reservoir level evolution. During most of the year, the reservoir level fluctuates between 0 and 1,200MWh. The maximum reservoir level is reached only once on the 1st of December 2015 although the level is not surpassed due to the price penalty.

Figure 6 - Reservoir level evolution



4.3 Model stress and back testing

The model has been tested in two ways. Firstly, stress tests have been carried out by changing values in the models parameters. In particular, the parameter L_i which is the indicator that the reservoir level is above the maximum level in hour i, has been set to 0 and also the penalty for breaching the maximum reservoir level has been set to 0. The second test was performed using actual historical spot prices and comparing the obtained values with those obtained for 2015.

4.3.1 Stress test

A stress test of the model was performed by changing the value of some of the parameters in the contract. In particular, the behaviour of the contracts was studied when setting the indicator of having reached the maximum reservoir level to 0 and setting the penalty to 0. Both model runs resulted in the same results which are described in the following paragraphs.

The results obtained for the first three quarters of 2015 did not vary and were exactly the same in the actual model run and in the stress testing runs. This is consistent as in the first three quarters of the year the maximum reservoir level is never reached. However, the results obtained in the fourth quarter of the year do vary as now there is no incentive to not surpass the maximum reservoir level. Indeed, the level of 3,000 is exceed. The maximum reservoir level reached is equal to 4,405.

The optimal dispatch of the contract results in a Profit of 1,549,628.90€, which is 1,925.69€ higher than in the actual contract dispatch. Therefore, the profit obtained from breaching the maximum reservoir level is minimal. This is due to the fact that the final reservoir level has to be 0.

From this stress test it can therefore be concluded that when the constraint of the maximum reservoir level is taken out, the optimal dispatch leads to the breaching of this maximum reservoir level however this doesn't translate into a noticeable increase in the profit obtained from the contract. Indeed, as can be deduced from the optimal dispatch from the first 3 quarters of the years, usually it is not profitable to reach very high reservoir levels but rather it is best to play with the peak to base ratio on a daily basis.

4.3.2 Back test

A back test was carried out using historical data. The aim of this back test is to determine whether the results obtained in the evaluation of the contract for 2015 are realistic and are in line with historical results. The model was back tested using actual spot prices of the second half of 2013 and the first half of 2014.

The optimal dispatch of the contract results in a Profit of 2,059,210.86€, which is 511,507.65€ higher than the obtained value for the contract in 2015.

Overall the profits obtained from the back test are 25% higher than those obtained from the model applied to 2015. This decrease in profits for 2015 is mainly due to two reasons. On the one hand it is due to the much lower level at which contracts for 2015 are trading compared to historical data. For example in the base the third quarter of 2013 delivered at 65.54 whilst the third quarter of 2015 was assessed at 51.35 on the 1st of July 2014. This is a decrease in value of the third quarter of the year of 22% between 2013 and 2015. Similarly, the fourth quarter of 2013 delivered at 65.18 whilst the fourth quarter of 2015 was assessed at 53.45 on the 1st of July 2014, being a decrease of 18% in the overall level of the quarter. It can therefore be deduced that the decrease of profits that would be obtained in 2015 is mainly explained by a decrease in prices.

The second reason behind this decrease in profits is the change in the daily profile of prices. In the past the hourly day shape of spot prices was more favourable for these types of contracts due to the higher peak to base ratio observed.

All in on all, the test results are positive and show that the obtained model results are coherent in their level and that their distribution throughout the year is coherent with the usual profit distribution that can be expected from these contracts.

5 EXTRINSIC VALUE: MULTISTAGE STOCHASTIC PROGRAMMING APPROACH

Uncertainty is one of the characteristics of most decision-making problems faced by electricity market participants. Decisions need to be made even with lack of perfect information and this is the main motivation for the use of stochastic programming models. A stochastic programming problem is an optimization problem where some of the variables are random and uncertain.

'If the input data of an optimization problem are well-defined and deterministic, its optimal solution (decision) is achieved by solving the problem. The decision is then implemented to attain the best outcome. However, more often than not, the input data are uncertain but describable through probability functions. In such a situation, it is not clear how the decision-making problem should be formulated. One possibility is to substitute the uncertain input data (describable through probability functions) by their corresponding expected values, which results in a well-defined and deterministic optimization problem. However, solving such a problem may lead to a solution that once implemented does not result in the best outcome' [Conejo]. The deterministic solution of the contract is thus equal to its intrinsic value. In order to evaluate the extrinsic value of the contract, a multistage stochastic programming approach can be followed.

6 CONCLUSIONS

The valuation of tolling agreements is a crucial and difficult task due to the highly volatile prices of commodities and in particular of electricity. The complexity of modeling price processes and of developing models capable of capturing actual market realities, makes the evaluation process a very interesting subject to study.

The motivation of this thesis was on the one hand to present tolling agreements and the different techniques that can be used to evaluate them, and on the other hand to present an actual example of the evaluation of a Hydro Virtual Pump Storage contract. In particular, one of the main objectives was to develop an HPFC builder and to implement it on a trading desk in order to assess different contracts offered in the market. This was achieved and the results obtained from it were considered very satisfactory. In fact, the robustness of the HPFC was confirmed by benchmarking against the HPFCs of Markit's Commodity Pricing Data [1], a service offered by an independent provider of forward curve, volatility and

[1] http://www.markit.com/Product/Pricing-Data-Commodities-Data

correlations data for the energy, metals and softs markets to support mark-to-market accounting.

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